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# Thermofield Dynamics of the Heterotic String
## — Thermal Cosmological Constant —

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## ABSTRACT

The thermofield dynamics of the $D = 10$ heterotic thermal string theory is exemplified at any finite temperature through the infrared behaviour of the one-loop cosmological constant in proper reference to the thermal duality symmetry in association with the global phase structure of the thermal string ensemble.

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Building up thermal string theories based upon the thermofield dynamics (TFD) has gradually been endeavoured in leaps and bounds. In the present communication, the TFD algorithm of the $D = 10$ heterotic thermal string theory is recapitulatively exemplified *à la* recent publication of ourselves [1], [2] through the infrared behaviour of the one-loop cosmological constant in proper respect of the thermal duality symmetry as well as the thermal stability of modular invariance. The global phase structure of the heterotic thermal string ensemble is also touched upon.

Let us start with the one-loop cosmological constant $\Lambda(\beta)$ as follows:

$$\Lambda(\beta) = \frac{\alpha'}{2} \lim_{\mu^2 \to 0} \mathrm{Tr} \left[ \int_{\infty}^{\mu^2} dm^2 \left( \Delta_B^\beta(p, P; m^2) + \Delta_F^\beta(p, P; m^2) \right) \right] \tag{1}$$

at any finite temperature $\beta^{-1} = kT$ in the $D = 10$ heterotic thermal string theory based upon the TFD algorithm, where $\alpha'$ means the slope parameter, $p^\mu$ reads loop momentum, $P^I$ lie on the root lattice $\Gamma_8 \times \Gamma_8$ for the exceptional group $E_8 \times E_8$ and the thermal propagator $\Delta_{B[F]}^\beta(p, P; m^2)$ of the free closed bosonic [fermionic] string is written *à la* Leblanc in the form

$$\begin{aligned} \Delta_{B[F]}^\beta(p, P; m^2) &= \int_{-\pi}^{\pi} \frac{d\phi}{4\pi} \, \mathrm{e}^{i\phi\left(N - \alpha - \bar{N} + \bar{\alpha} - 1/2 \cdot \sum_{I=1}^{16} (P^I)^2\right)} \\ &\times \left( \left[ \underset{[\,-\,]}{+} \int_0^1 dx + \frac{1}{2} \sum_{n=0}^{\infty} \frac{\delta[\alpha'/2 \cdot p^2 + \alpha'/2 \cdot m^2 + 2(n - \alpha)]}{\mathrm{e}^{\beta|p_0|} \underset{[\,+\,]}{-} 1} \oint_c dx \right] \right. \\ &\left. \times x^{\alpha'/2 \cdot p^2 + N - \alpha + \bar{N} - \bar{\alpha} + 1/2 \cdot \sum_{I=1}^{16} (P^I)^2 + \alpha'/2 \cdot m^2 - 1} \right) \quad , \end{aligned} \tag{2}$$

where $N$ [$\bar{N}$] denotes the number operator of the right- [left-] mover, the intercept parameter $\alpha$ [$\bar{\alpha}$] is fixed at $\alpha = 0$ [$\bar{\alpha} = 1$] and the contour $c$ is taken as the unit circle around the origin. The modular parameter integral representation of $\Lambda(\beta)$ is then written *à la* O'Brien and Tan as follows:

$$\Lambda(\beta) = -8(2\pi\alpha')^{-D/2}\int_E \frac{d^2\tau}{2\pi\tau_2^2}\,(2\pi\tau_2)^{-(D-2)/2}\,\mathrm{e}^{2\pi i\bar{\tau}}\left[1+480\sum_{m=1}^{\infty}\sigma_7(m)\bar{z}^m\right]$$

$$\times\prod_{n=1}^{\infty}(1-\bar{z}^n)^{-D-14}\left(\frac{1+z^n}{1-z^n}\right)^{D-2}\sum_{\ell\in Z;\mathrm{odd}}\exp\left[-\frac{\beta^2}{4\pi\alpha'\tau_2}\,\ell^2\right]\;;\quad D=10\quad, \tag{3}$$

where $\overset{[-]}{\tau} = \tau_1 \underset{[-]}{+} i\tau_2$, $z = x\mathrm{e}^{i\phi} = \mathrm{e}^{2\pi i\tau}$, $\bar{z} = x\mathrm{e}^{-i\phi} = \mathrm{e}^{-2\pi i\bar{\tau}}$, $E$ means the half-strip region in the $\tau$ plane, *i.e.* $-1/2 \leq \tau_1 \leq 1/2$; $\tau_2 > 0$. The "$E$-type" thermal amplitude $\Lambda(\beta)$ obtained above is not modular invariant and annoyed with ultraviolet divergences for $\beta \leq \beta_H = (2+\sqrt{2})\pi\sqrt{\alpha'}$, where $\beta_H$ reads the inverse Hagedorn temperature of the heterotic thermal string.

Our prime concern is reduced to regularizing the thermal amplitude $\Lambda(\beta)$ *à la* O'Brien and Tan through transforming the physical information in the ultraviolet region of the half-strip $E$ into the "new-fashioned" modular invariant amplitude. Let us postulate the one-loop dual symmetric thermal cosmological constant $\bar{\Lambda}(\beta; D)$ at any space-time dimension $D$ as an integral over the fundamental domain $F$ of the modular group $SL(2,Z)$ as follows:

$$\begin{aligned}\bar{\Lambda}(\beta;D) \;=\;& \frac{2}{\beta}(2\pi\alpha')^{-D/2}\sum_{(\sigma,\rho)}\int_F \frac{d^2\tau}{2\pi\tau_2^2}\,(2\pi\tau_2)^{-(D-2)/2}\,\bar{z}^{-(D+14)/24}z^{-(D-2)/24}\\ &\times\left[1+480\sum_{m=1}^{\infty}\sigma_7(m)\bar{z}^m\right]\prod_{n=1}^{\infty}(1-\bar{z}^n)^{-D-14}(1-z^n)^{-D+2}\\ &\times A_{\sigma\rho}(\tau;D)\left[C_\sigma^{(+)}(\bar{\tau},\tau;\beta)+\rho C_\sigma^{(-)}(\bar{\tau},\tau;\beta)\right]\quad,\end{aligned} \tag{4}$$

where

$$\begin{pmatrix} A_{+-}(\tau;D)\\ A_{-+}(\tau;D)\\ A_{--}(\tau;D)\end{pmatrix} = 8\left(\frac{\pi}{4}\right)^{(D-2)/6}\begin{pmatrix} -[\theta_2(0,\tau)/\theta_1'(0,\tau)^{1/3}]^{(D-2)/2}\\ -[\theta_4(0,\tau)/\theta_1'(0,\tau)^{1/3}]^{(D-2)/2}\\ [\theta_3(0,\tau)/\theta_1'(0,\tau)^{1/3}]^{(D-2)/2}\end{pmatrix}\quad, \tag{5}$$

$$C_{\sigma}^{(\gamma)}(\bar{\tau},\tau;\beta) = (4\pi^2\alpha'\tau_2)^{1/2} \sum_{(p,q)} \exp\left[-\frac{\pi}{2}\left(\frac{\beta^2}{2\pi^2\alpha'}p^2 + \frac{2\pi^2\alpha'}{\beta^2}q^2\right)\tau_2 + i\pi pq\tau_1\right], \tag{6}$$

the signatures $\sigma, \rho$ and $\gamma$ read $\sigma, \rho = +,-;\ -,+;\ -,-$ and $\gamma = +,-$, respectively, and the summation over $p$ $[q]$ is restricted by $(-1)^p = \sigma$ $[(-1)^q = \gamma]$. It is almost needless to mention that $\bar{\Lambda}(\beta; D = 10)$ is literally identical with $\Lambda(\beta)$. The thermal amplitude $\bar{\Lambda}(\beta; D)$ is manifestly modular invariant and free of ultraviolet divergences for any value of $\beta$ and $D$. If and only if $D = 10$, in addition, the thermal duality relation $\beta\bar{\Lambda}(\beta; D) = \tilde{\beta}\bar{\Lambda}(\tilde{\beta}; D)$ is manifestly satisfied for the thermal amplitude $\bar{\Lambda}(\beta; D)$, irrespective of the value of $\beta$, where $\tilde{\beta} = 2\pi^2\alpha'/\beta$.

The infrared behaviour of the thermal cosmological constant $\bar{\Lambda}(\beta; D)$ is asymptotically described as

$$\begin{aligned}\bar{\Lambda}(\beta; D) \quad &= \quad \frac{2}{\beta}\,(4\pi^2\alpha')^{-D/2}\int_F d^2\tau\ \tau_2^{-(D+2)/2}\exp\left[\pi\tau_2\cdot\frac{D+6}{6}\right] \\ &\quad \times\left[A_{-+}(\tau; D) - A_{--}(\tau; D)\right] C_{-}^{(-)}(\bar{\tau},\tau;\beta) \quad .\end{aligned} \tag{7}$$

The $D = 10$ TFD amplitude $\bar{\Lambda}(\beta; D = 10)$ is then infrared divergent for $(2-\sqrt{2})\pi\sqrt{\alpha'} = \tilde{\beta}_H \leq \beta \leq \beta_H$, where $\tilde{\beta}_H$ reads the inverse dual Hagedorn temperature of the heterotic thermal string. We can therefore define the dimensionally regularized, $D = 10$ one-loop dual symmetric thermal cosmological constant $\hat{\Lambda}(\beta)$ in the sense of analytic continuation from $D < 2/5$ to $D = 10$ by

$$\begin{aligned}\hat{\Lambda}(\beta) = &-\frac{2}{\beta}(8\pi\alpha')^{-(D-1)/2}\sum_{(p,q)}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \exp[i\pi pq\tau_1] \\ &\times\left(\frac{\beta^2}{2\pi^2\alpha'}\,p^2 + \frac{2\pi^2\alpha'}{\beta^2}\,q^2 - 6 - i\varepsilon\right)^{(D-1)/2} \\ &\times\Gamma\left[-\frac{D-1}{2}\,,\ \frac{\pi}{2}\sqrt{1-\tau_1^2}\left(\frac{\beta^2}{2\pi^2\alpha'}\,p^2 + \frac{2\pi^2\alpha'}{\beta^2}\,q^2 - 6 - i\varepsilon\right)\right] \quad ; \quad D = 10 \quad ,\end{aligned} \tag{8}$$

irrespective of the value of $\beta$, where $p, q = \pm 1; \pm 3; \pm 5; \cdots$, and $\Gamma$ is the incomplete gamma function of the second kind.

The dimensionally regularized, thermal cosmological constant $\hat{\Lambda}(\beta)$ manifestly satisfies the thermal duality relation $\beta\hat{\Lambda}(\beta) = \tilde{\beta}\hat{\Lambda}(\tilde{\beta})$ in full accordance with the thermal stability of modular invariance. The thermal duality symmetry immediately yields the asymptotic formula as follows: $\hat{\Lambda}(\beta \sim 0) \sim 2\pi^2\alpha'/\beta^2 \cdot \hat{\Lambda}(\beta^{-1} \to 0) = 2\pi^2\alpha'/\beta^2 \cdot \Lambda$ for the heterotic thermal string theory, where $\Lambda$ literally reads the $D = 10$ zero-temperature, one-loop cosmological constant which is in turn guaranteed to vanish automatically as an inevitable consequence of the Jacobi identity $\theta_2^4 - \theta_3^4 + \theta_4^4 = 0$ for the theta functions. The present observation is paraphrased *à la* Osorio as follows: The thermal duality symmetry is inherent to the fact that the total number of degrees of freedom vanishes at extremely high temperature $\beta \sim 0$ in the sense of the modular invariant counting. Accordingly, it seems possible to claim *à la* Atick and Witten that the heterotic thermal string will be asymptotically described at high temperature by underlying topological theory. The present view may deserve more than passing consideration in an attempt to substantialize the geometrical ideas purely topological in character.

Let us describe the singularity structure of the dimensionally regularized, dual symmetric thermal amplitude $\hat{\Lambda}(\beta)$. The position of the singularity $\beta_{|p|,|q|}$ is determined by solving $\beta/\tilde{\beta} \cdot p^2 + \tilde{\beta}/\beta \cdot q^2 - 6 = 0$ for every allowed $(p, q)$ in eq. (8). Thus we obtain a set of solutions as follows: $\beta_{1,1} = \beta_H = (\sqrt{2} + 1)\pi\sqrt{2\alpha'}$ and $\tilde{\beta}_{1,1} = \tilde{\beta}_H = (\sqrt{2} - 1)\pi\sqrt{2\alpha'}$ which form the leading branch points of the square root type at $\beta_H$ and $\tilde{\beta}_H$, respectively. We are now in the position to touch upon the global phase structure of the heterotic thermal string ensemble. There will then exist three phases in the sense of the thermal duality symmetry as follows: (i) the $\beta$ channel canonical phase in the tachyon-free region $(2 + \sqrt{2})\pi\sqrt{\alpha'} = \beta_H \leq \beta < \infty$, (ii) the dual $\tilde{\beta}$ channel canonical phase in the tachyon-free region $0 < \beta \leq \tilde{\beta}_H = (2 - \sqrt{2})\pi\sqrt{\alpha'}$ and (iii) the self-dual microcanonical phase in the tachyonic region $\tilde{\beta}_H < \beta < \beta_H$. There will appear no effective splitting of the

microcanonical domain because of the absence of the self-dual leading branch point at $\beta_0 = \tilde{\beta}_0 = \pi\sqrt{2\alpha'}$. As a consequence, it still remains to be clarified whether the so-called maximum temperature of the heterotic string excitation is asymptotically described as $\beta_0^{-1} = \tilde{\beta}_0^{-1}$ in proper respect to the self-duality of the microcanonical phase.

We have succeeded in shedding some light upon physical aspects of the thermal duality symmetry in full harmony with the thermal stability of modular invariance through the infrared behaviour of the one-loop cosmological constant for the dimensionally regularized, $D = 10$ heterotic thermal string theory based upon the TFD algorithm.

# References


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